Faster Algorithms for Minimum-Energy Scheduling of Wireless Data Transmissions

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Abstract

This paper considers the problem of minimizing the energy used in transmitting a given sequence of packets with specified completion deadlines from a node in a wireless packet network. The packets have to to be transmitted in first-in-first-out order. Packets can be destined to different receivers. The channel conditions to each receiver (and so the energy per bit needed) can be different and assumed to be known. An offline algorithm for the case where all the packets have a common completion deadline was presented in [3, 4], and was used as the basis for an online scheduling algorithm. In this paper, we present a faster offline algorithm for the more general case of different packet completion deadlines. The presented algorithm has a running time of $O(M^2)$, where M is the length of the given packet sequence, provided the inverse of the derivative of the energy function is invertible in closed form. Otherwise, the running time depends on the number of receivers as well.

1 Introduction

The need for extending lifetimes in sensor networks and the operating time before recharging in wireless devices has led to energy efficiency of protocols and battery management becoming important issues in sensor and wireless networking research. Several recent papers have considered the issue of extending lifetimes of sensor networks by energy efficient routing [5, 7]. Transmission power control schemes have been extensively studied. Here the objective is mostly to maximize the amount of information sent subject to specified average power use constraints. Energy efficiency, the ratio of total data delivered to total energy used, was considered in [6] where the energy efficiency of MAC protocols was studied. The problem of energy efficiency in the transmission of a sequence of packets has been receiving increasing attention [2, 3, 4].

Minimizing the total amount of energy consumed for transmitting, in first-in-first-out order, a given sequence of packets subject to a common deadline was studied in [3, 4]. The problem was formulated as a convex optimization problem and an iterative algorithm that converges to the optimal solution, but is not polynomially bounded in the length of the packet sequence to be transmitted, was presented.

In this paper, we formulate and solve the more general problem of minimum energy transmission of packets with individual packet deadlines, as a convex optimization problem over a linear polyhedron with a special structure. We exploit this structure, to develop polynomial time algorithms for

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solving the problem for a fairly general class of energy functions. We also outline how the algorithms can be used in the case where the energy functions do not fall in this class. The algorithms are almost greedy and are very simple to implement. The approach can be adapted to the case where the objective is not the total energy consumed but is to minimize the maximum power used during transmissions.

2 Problem Formulation

Here, we outline the general problem of minimizing the energy with deadlines on the individual packets. The problem is an offline problem in the sense that all the packet arrival times and the packet deadlines are assumed to be known ahead of time. These can form the basis for on-line algorithms with good performance.

2.1 Notations and Assumptions

Most notations follow that used in [3, 4]. Consider a wireless channel with a single transmitter node and multiple receivers. Assume that M packets arrive to the transmitter node at times t_i in the interval $[0, T]$, with $0 = t_1 < t_2 < ... < t_M < T$. After receiving each packet i and buffering it, the node starts transmitting it at time s_i for a duration of τ_i, until finish time $f_i = s_i + \tau_i$.

We assume four constraints on the packet transmission times. First, since the transmitter node needs to receive a packet before sending it, $t_i \leq s_i$. Second, the start time of any transmission needs to precede its finish time, and thus $s_i \leq f_i$. Third, for each packet i, we introduce a deadline time $d_i \in [0, T]$ on the finish time of the packet transmission, such that $f_i \leq d_i$. This deadline time will be used in order to model such constraints as a finite buffer size and real-time traffic requirements. Finally, we assume that the queue is FIFO (First In First Out), and hence that packets are sent in the order in which they were received, with at most one packet sent at any given time. As a summary of these constraints, we thus assume that for each packet i:

$$0 \leq t_i \leq s_i \leq f_i \leq d_i \leq T, \tag{1}$$

and that the transmitter schedule satisfies:

$$0 \leq s_1 \leq f_1 \leq s_2 \leq f_2 \leq ... \leq s_M \leq f_M \leq T. \tag{2}$$

A packet transmission schedule is feasible if it satisfies these two constraint equations.

Given these scheduling constraints, the objective of the packet scheduler is to minimize the total energy needed to transmit the M packets. Let $w_i(\tau_i)$ be the energy needed to transmit the i^{th} packet in a duration of $\tau_i \geq 0$. We assume that w_i is twice differentiable, with $w_i(\tau_i) > 0$, $\dot{w}_i(\tau_i) < 0$ and $\ddot{w}_i(\tau_i) > 0$. In other words, the energy needed to send packet i in time τ_i is positive, decreasing with τ_i and strictly convex. These assumptions are justified in [3]. Then the objective of the packet scheduler is to find a feasible schedule for the start and finish transmission times of each packet such that the total energy $\sum_{i=1}^{k} w_i(\tau_i)$ is minimized. Thus, the energy minimization problem can be formulated as:

$$\begin{aligned} \min \quad & \sum_{i=1}^{k} w_i(\tau_i) && (3)\\ \text{s.t.: } f_i \quad = \quad & s_i + \tau_i \quad \forall i, && (4)\\ 0 \quad \leq \quad & t_i \leq s_i \leq f_i \leq d_i \leq T \quad \forall i, && (5)\\ 0 \quad \leq \quad & s_1 \leq f_1 \leq ... \leq s_M \leq f_M \leq T. && (6) \end{aligned}$$

3 Scheduling with Common-Deadlines

When all packets share a common deadline T, the optimization problem in Equations (4)-(6) simplifies into the following problem. Note that we do not assume that the objective functions $w_i()$ are identical for all i. In other words, we assume that the packets are transmitted to different users with different energy characteristics.

$$\min \sum_{i=1}^{k} w_i(\tau_i)$$

$$\begin{aligned} \text{s.t.:} \sum_{j=i}^{M} \tau_j &\leq T - t_i \quad i = M, M-1, \ldots, 2 \\ \sum_{j=1}^{M} \tau_j &= T, \\ \tau_i &\geq 0 \quad \forall i. \end{aligned}$$

We now outline the algorithm COMMON_DEADLINE to solve this problem optimally. In the description of the algorithm, we use $\dot{w}_i^{-1}()$ to represent the inverse of the derivative function.

Theorem 1 *Algorithm COMMON_DEADLINE computes the optimal scheduling times for the packets for the common deadline problem.*

Proof: (Outline)
We first note that the common deadline problem is a separable convex minimization problem. We write the Karush-Kuhn-Tucker (KKT) conditions [1] for optimality. We associate dual multipliers $\theta_M \leq 0$ with the first constraint, $\theta_{M-1} \leq 0$ with the second constraint, and so on and θ_1 with the last constraint. Since the last constraint is an equality, θ_1 is unrestricted in sign. The KKT conditions are

$$\dot{w}_j^{-1}(\tau_j^*) = \sum_{k=1}^{k} \theta_k \qquad j = M, M-1, \ldots, 1$$

where τ_j^* represents the optimal primal solution. In addition, we have the complementary slackness conditions that imply that if $\sum_{j=k}^{M} \tau_j^* < T - t_k$ then $\theta_k = 0$. Since the objective function is convex, the KKT conditions are necessary and sufficient for optimality. Let $\alpha_j = \min_{i \leq j} \gamma_i$ and let $\theta_1 = \alpha_1$ and $\theta_j = \alpha_j - \alpha_{j-1}$. Note that by construction $\theta_j \leq 0$ and

$$\dot{w}_j^{-1}(\tau_j^*) = \sum_{k=1}^{j} \theta_k \qquad j = M, M-1, \ldots, 1.$$

Further note that complementary slackness automatically holds. $\diamond$

In the case where all the energy functions are identical, the algorithm reduces to the algorithm given in [3]. The algorithm is substantially simpler than the move-to-right algorithm given in [4].

Initialize $u = M + 1$, $A = 0$ and $\gamma_i = \infty$ $1 \le i \le M$.
While $u > 1$
For $k = 1, 2, \ldots, u - 1$
Solve for λ_k in
$\sum_{j=k}^{u-1} \dot{w_j}^{-1}(\lambda_k) = T - t_k - A$
end For
Let $u' =$ arg $\min_{1 \le k \le u-1} \lambda_k$
Let $\gamma_{u'} = \min_{1 \le k \le u-1} \lambda_k$.
Let $A = T - \tau_{u'}$
$\tau_j^* = \dot{w_j}^{-1}(\gamma_u')$ $j = u+1, u+2, \ldots, u'$
Set $u = u'$.
end While

3.1 Some Implementation Details

The running time of the algorithm depends on determining the solution to

$$\sum_{j=k}^{u-1} \dot{w_j}^{-1}(\lambda_k) = T - t_k - A$$

This depends on the functional form of $\dot{w_j}^{-1}()$. If the function is invertible in closed form, then the overall running time of the algorithm is $O(M^2)$. If the function is not invertible in closed form the approximations can be used to solve the problem in closed form. For example, in the case of optimal channel coding over an Additive White Gaussian Noise (AWGN) channel with noise power N_i and packet length L_i yields:

$$w_i(\tau_i) = \tau_i N_i (2^{\frac{2L_i}{\tau_i}} - 1).$$

If we write a Taylor series expansion for the function, we get:

$$\begin{aligned} w_i(\tau_i) &= \tau_i N_i \left(\frac{2L_i \ln 2}{\tau_i} + \frac{4{L_i}^2 (\ln 2)^2}{{\tau_i}^2} + O\left(\frac{1}{{\tau_i}^3}\right)\right) \\ &= 2L_i N_i \ln 2 + \frac{4{L_i}^2 N_i (\ln 2)^2}{\tau_i} + O\left(\frac{1}{{\tau_i}^2}\right) \\ &= A_i + \frac{B_i}{\tau_i} + O\left(\frac{1}{{\tau_i}^2}\right), \end{aligned}$$

where A_i and B_i are two positive factors depending only on i. With this approximation, note that the inverse can be computed in closed form and hence the overall running time of the algorithm is $O(M^2)$. In the case where approximations are not possible, it is easy to show that the solution of each equation can be determined by one dimensional searches. The complexity grows as a function of the number of receivers.

Packet i	N_i	t_i	A_i	B_i
1	1	0.0	0.0138	0.0002
2	6	0.2	0.0832	0.00114
3	2	0.3	0.0277	0.0004
4	4	0.8	0.0555	0.0008

Table 1: Problem Parameters

3.2 Numerical Example

We now illustrate the algorithm for the common deadline case using a numerical example. The objective of the example is to give a slightly different and intuitive interpretation of the algorithm. Assume that we have a system with 4 packets that have to be transmitted. We use one of the energy functions used in [4]. We assume that the packets are transmitted to four different receivers with different noise powers. (One can equivalently assume that different receivers have different channel gains). We assume that the channels are AWGN with a symbol rate of 10^6 transmissions/s with noise power N_i to receiver i (or equivalently for transmission of packet i since for this example we assume that each packet is transmitted to a different receiver). Packet sizes are taken to be 10kB. The energy function for each packet i's transmission is then given by

$$w(\tau_i) = 10^6 N_i \tau_i \left(2^{0.02\tau_i^{-1}} - 1\right).$$

The noise level N_i for the different packets is shown in Table 1. The arrival time t_i of the packets is shown in Table 1 and we assume that $T = 1$. The problem can now be written as

$$\min \sum_{i=1}^{4} w_i(\tau_i)$$

$$\begin{aligned} \tau_4 &\leq 1 - 0.8 = 0.2 \\ \tau_3 + \tau_4 &\leq 1 - 0.3 = 0.7 \\ \tau_2 + \tau_3 + \tau_4 &\leq 1 - 0.2 = 0.8 \\ \tau_1 + \tau_2 + \tau_3 + \tau_4 &= 1 \end{aligned}$$

Associating dual multipliers of $\theta_1, \theta_2, \theta_3 \leq 0$ with the first three inequalities respectively and θ_4 with the last equation, the KKT conditions are given by

$$\begin{aligned} \dot{w_1}^{-1}(\tau_1^*) &= \theta_1 \\ \dot{w_2}^{-1}(\tau_2^*) &= \theta_1 + \theta_2 \\ \dot{w_3}^{-1}(\tau_3^*) &= \theta_1 + \theta_2 + \theta_3 \\ \dot{w_4}^{-1}(\tau_4^*) &= \theta_1 + \theta_2 + \theta_3 + \theta_4 \end{aligned}$$

In addition, we have the following complementary slackness conditions

$$\begin{aligned} \theta_4 \left(\tau_4 - 0.2\right) &= 0 \\ \theta_3 \left(\tau_3 + \tau_4 - 0.7\right) &= 0 \\ \theta_2 \left(\tau_2 + \tau_3 + \tau_4 - 0.8\right) &= 0 \\ \theta_1 \left(\tau_1 + \tau_2 + \tau_3 + \tau_4 - 1\right) &= 0 \end{aligned}$$

We use the approximation to the AWGN channel described in the last section and we determine the parameters A_i and B_i in Table 1. For simplicity, we ignore the factor 10^6 in the illustrative example. Recall that $A_i = N_i(0.02 \ln 2)$, $B_i = N_i(0.02 \ln 2)^2$, and

$$w(\tau_i) = A_i + \frac{B_i}{\tau_i} \qquad i = 1, 2, 3, 4$$

therefore

$$\dot{w}_i = \frac{-B_i}{\tau_i^2} \quad i = 1, 2, 3, 4.$$

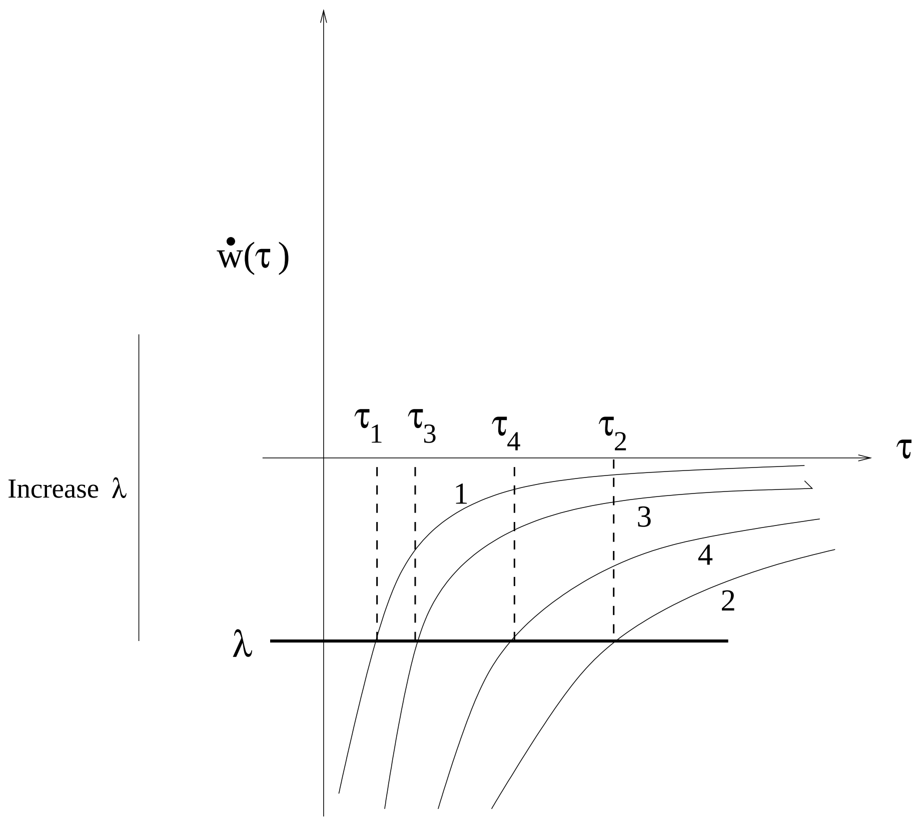

Figure 1: Algorithm COMMON_DEADLINE : Illustration

Figure 1, shows the plot of $\dot{w}(\tau)$ with respect to τ. In Figure 1, λ represents the parameter that we vary during the course of the algorithm. The value of λ represents the value of the derivative of the objective function. Since the functional form is invertible, if $\dot{w_i}() = \lambda$ then

$$\tau_i = \sqrt{\frac{-B_i}{\lambda}}.$$

The algorithm starts of by setting the value of $\lambda = -\infty$ and hence the value of all the $\tau_i = 0$. We now increase the value of λ until exactly one of the constraints is binding. This can be computed easily since the value of τ_i is known as a function of λ. The smallest value of λ at which one of the constraints becomes binding can be computed as follows: (Recall here that λ is negative due to the class of energy functions considered.)

$$\sqrt{-\lambda} = \max\{\frac{\sqrt{-B_4}}{0.2}, \frac{\sqrt{-B_4}+\sqrt{-B_3}}{0.7}, \frac{\sqrt{-B_4}+\sqrt{-B_3}+\sqrt{-B_2}}{0.8}, \frac{\sqrt{-B_4}+\sqrt{-B_3}+\sqrt{-B_2}+\sqrt{-B_1}}{1.0}\}.$$

The constraint that is most binding is $\tau_4 \leq 0.2$ at the value of $\lambda = -0.0199$. (This value is represented by γ_4 in the algorithm description). The value of τ_4 is fixed at $\tau_4^* = 0.2$ and eliminated from the problem leaving:

$$\begin{aligned} \tau_3 &\leq 0.5 \\ \tau_2 + \tau_3 &\leq 0.6 \\ \tau_1 + \tau_2 + \tau_3 &= 0.8 \end{aligned}$$

We now continue to increase the value of λ. The constraint that is binding next is $\tau_2 + \tau_3 \leq 0.6$ at $\lambda = -0.008$. This value is represented as γ_2 in the algorithm description. The optimal values for $\tau_3^* = 0.22$ and $\tau_2^* = 0.38$ and these two variables are eliminated from the problem leaving only the constraint $\tau_1 = 0.2$. This constraint is binding at $\lambda = \gamma_1 = -0.005$. Note that setting $\theta_1 = \gamma_1, \theta_2 = \gamma_2 - \gamma_1, \theta_3 = 0$ and $\theta_4 = \gamma_4 - \gamma_2$ verifies optimality. (Note that the KKT conditions including complementary slackness are satisfied by the primal-dual pair). If the derivative of the objective function is not invertible, the equations have to be solved numerically. Essentially the same idea works in the case of individual packet deadlines as outlined in the next section. In that case, instead of checking n equations and inequalities in each step, we have to check $2n-1$ inequalities and equation at each step of the algorithm. Though the description of the algorithm is more involved, the key ideas behind the algorithm is the same as the algorithm for individual packet deadlines.

4 Scheduling with Individual Packet Deadlines

In this section, we consider the most general case, where each packet has its own deadline. We show that an algorithm very similar in spirit to the problem with a common deadline solves the problem optimally. The algorithm PACKET_DEADLINE is shown below:

```
Initialize $K = M$, $l = 0$ , $u = M + 1$
$\mathcal{N} = \{1, 2, \ldots, n\}$, $\mathcal{F} = \mathcal{N}$, $LS = 0$, $US = 0$.
While $\mathcal{F} \neq \emptyset$
    For $n = l + 1, l + 2, \ldots, u - 2$
        Solve for $\lambda_n$ in
        $\sum_{j=l+1}^{n} \dot{w_j}^{-1}(\lambda_n) = d_j - LS$
    For $n = l + 2, l + 3, \ldots, u - 1$
        Solve for $\delta_n$ in
        $\sum_{j=n}^{u-1} \dot{w_j}^{-1}(\delta_n) = T - t_n - US$
    Solve for $\phi$ in
        $\sum_{j=l+1}^{u-1} \dot{w_j}^{-1}(\phi) = T - LS - US$
    Let $\omega = \min\{\min_n\{\lambda_n, \theta_n\}, \phi\}$
    If $\omega = \lambda_n$ for some $n$ then
        Set $\gamma_n = \omega$
        Set $\tau_j^* = \dot{w_j}^{-1}(\omega)$ for $l + 1 \leq j \leq n$.
        Update $LS = LS + \sum_{i=l+1}^{n} \tau_i$
        Set $l = n$ and $\mathcal{L} = \{1, 2, \ldots n\}$
        Set $\mathcal{F} = \mathcal{N} \backslash \mathcal{L} \backslash \mathcal{U}$.
    If $\omega = \delta_n$ for some $n$ then
        Set $\gamma_n = \omega$.
        Set $\tau_j^* = \dot{w_j}^{-1}(\omega)$ for $n \leq j \leq u - 1$.
        Update $US = US + \sum_{i=n}^{u-1} \tau_i$
        Set $u = n$ and $\mathcal{U} = \{n, n + 1, \ldots M\}$
        Set $\mathcal{F} = \mathcal{N} \backslash \mathcal{L} \backslash \mathcal{U}$.
    If $\omega = \phi$ for some then
        $\tau_j^* = \dot{w_j}^{-1}(\omega)$ for $l + 1 \leq j \leq u - 1$.
        Set $F = \emptyset$
end While
```

Uisng a proof technique similar to the COMMON_DEADLINE algorithm, we can show the following result.

Theorem 2 *The algorithm PACKET_DEADLINE computes the optimal scheduling times for the packets for the individual packet deadline problem and the minimum energy transmission times are given by the vector* τ^*.

In the case where the objective is to minimize the maximum power consumed, the algorithm to solve this problem is similar to PACKET_DEADLINE where the energy function is used wherever the derivative of the energy function is used in the PACKET_DEADLINE algorithm.

5 Conclusions

We presented polynomial time offline algorithms for the minimum energy transmission of a sequence of packets subject to deadlines. The algorithms are almost greedy in nature and can be adapted easily to get heuristics for the solution of dynamic problems where the arrival of packets is random and is not known ahead of time.

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